Spherical (De)Convolution for Inverse Rendering

Ravi Ramamoorthi

Columbia Vision and Graphics Center
Columbia University

Workshop on Inverse Problems at Columbia: May 3, 2007

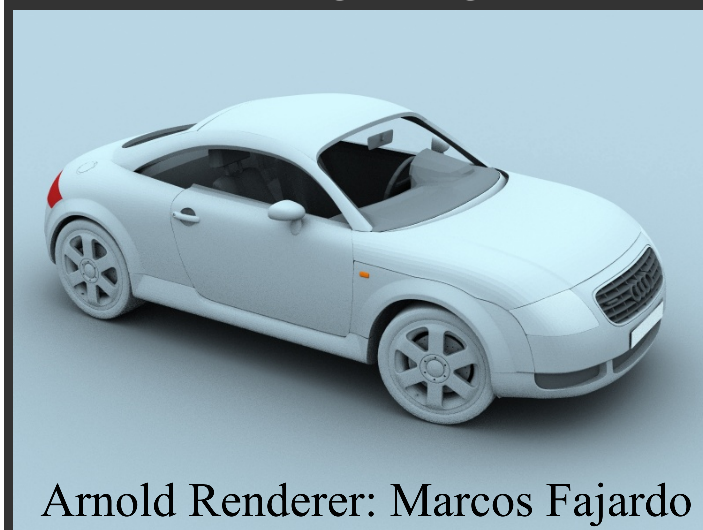
Photorealistic Rendering

Geometry

70’s, 80’s: Splines

90’s: Range Data →

Rendering Algorithm



80’s,90’s: Physically based

Materials/Lighting

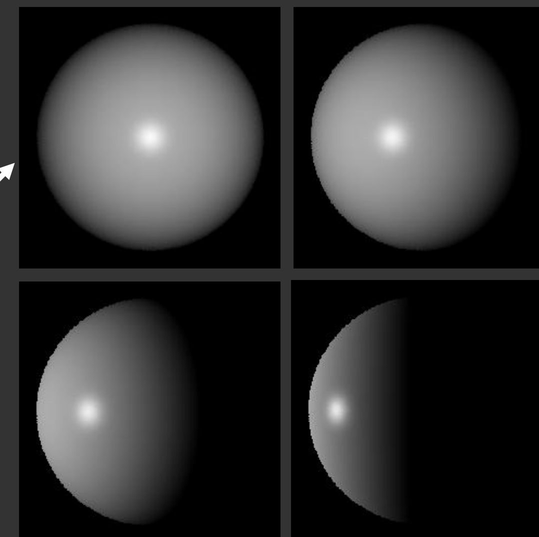
(Texture Reflectance[BRDF] Lighting)

Realistic input models required

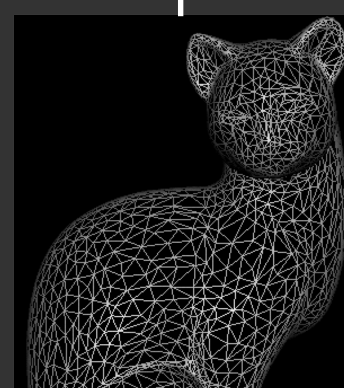
Inverse Rendering

Photographs

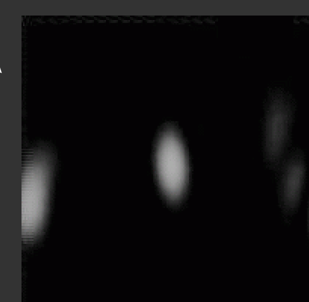
Inverse Rendering Algorithm



BRDF

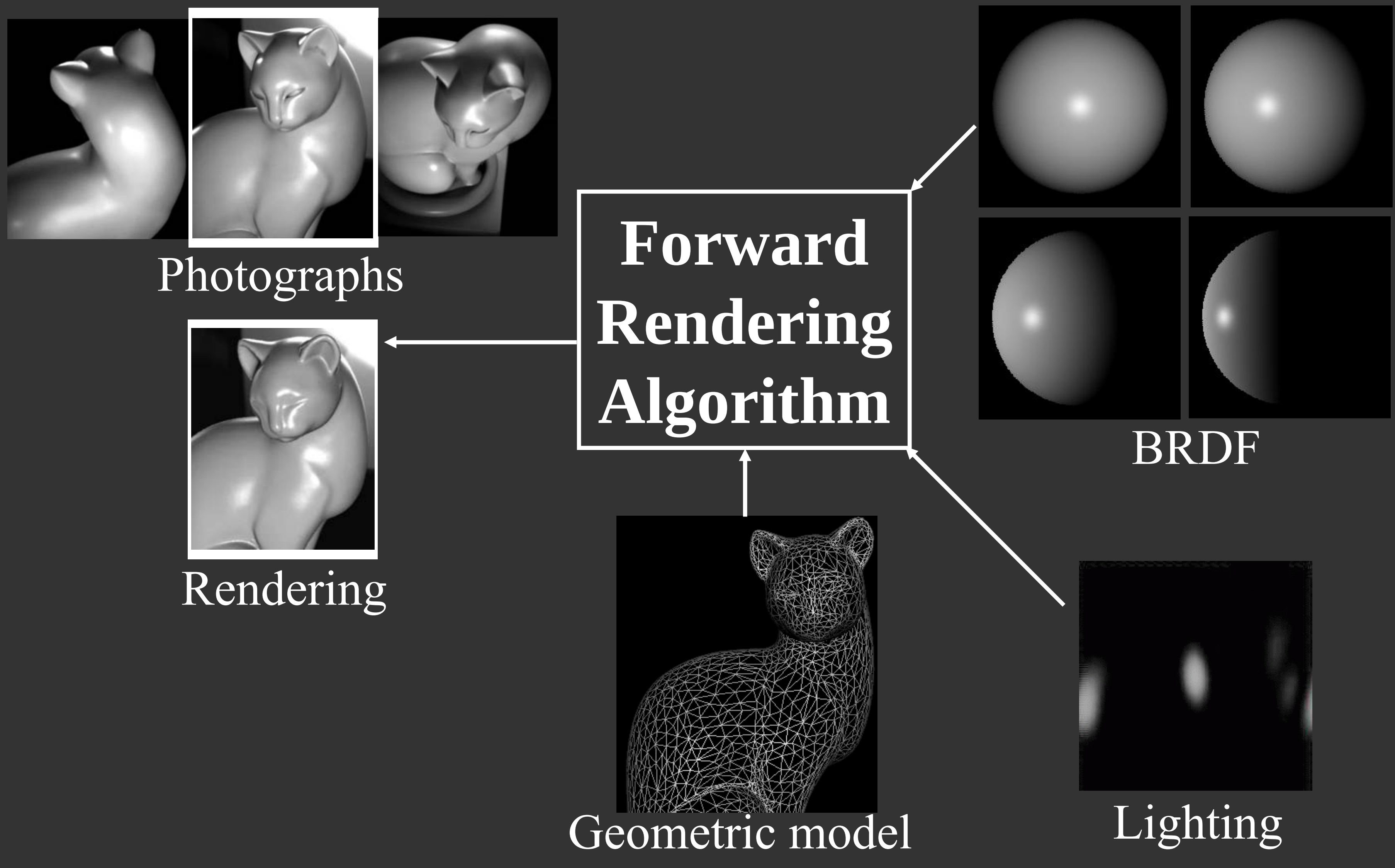


Geometric model

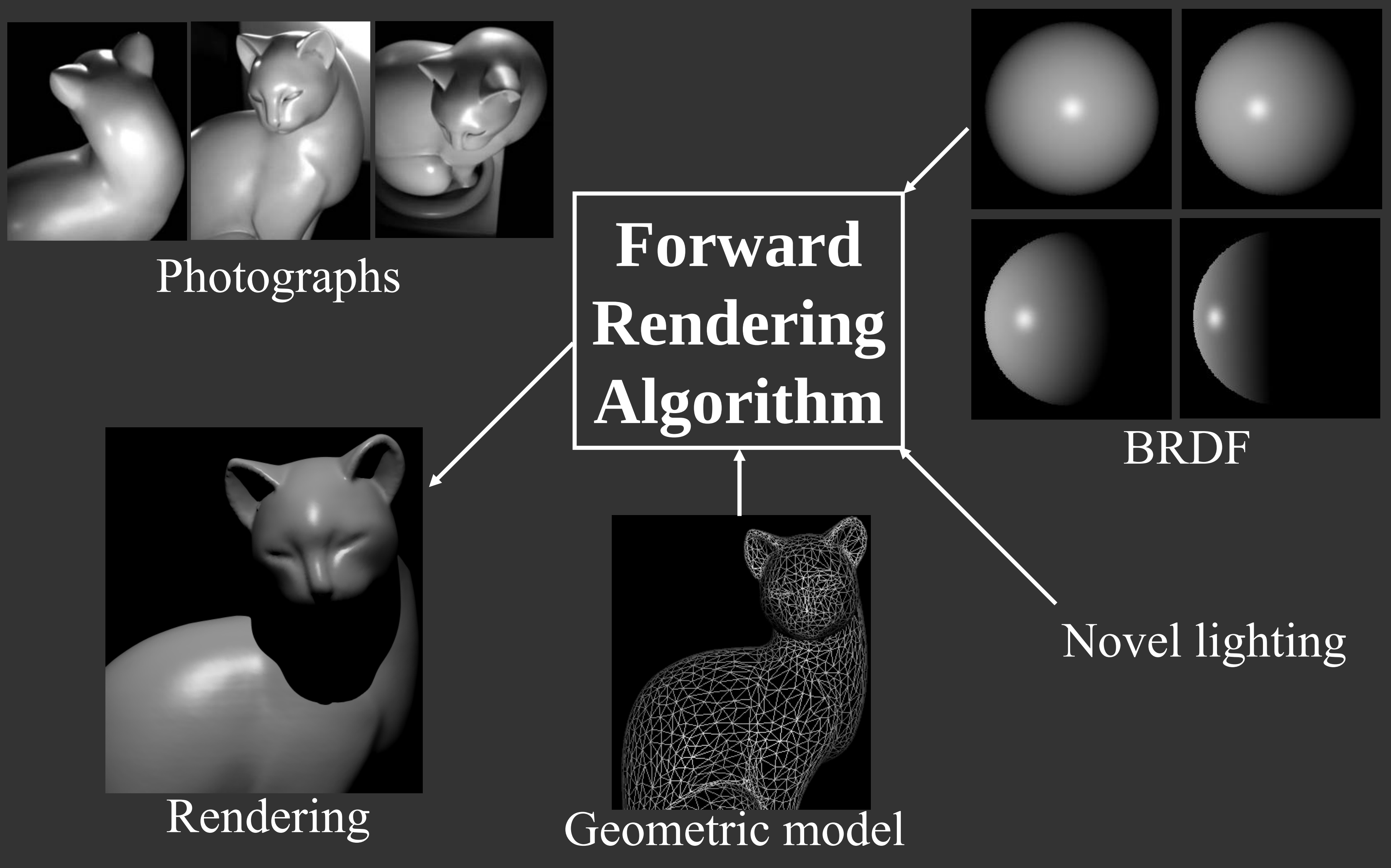


Lighting

Inverse Rendering

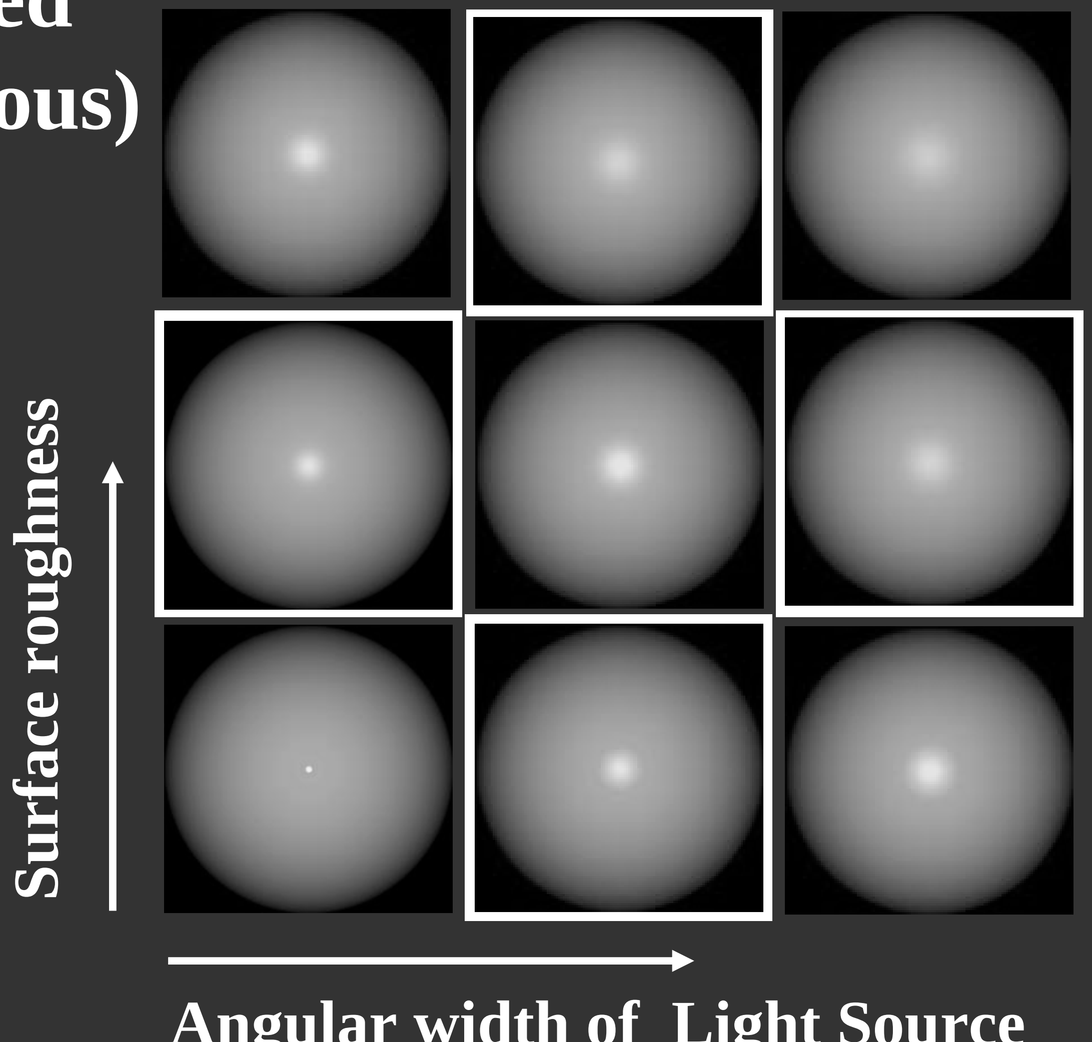


Inverse Rendering



Inverse Problems: Difficulties

Ill-posed (ambiguous)



Outline

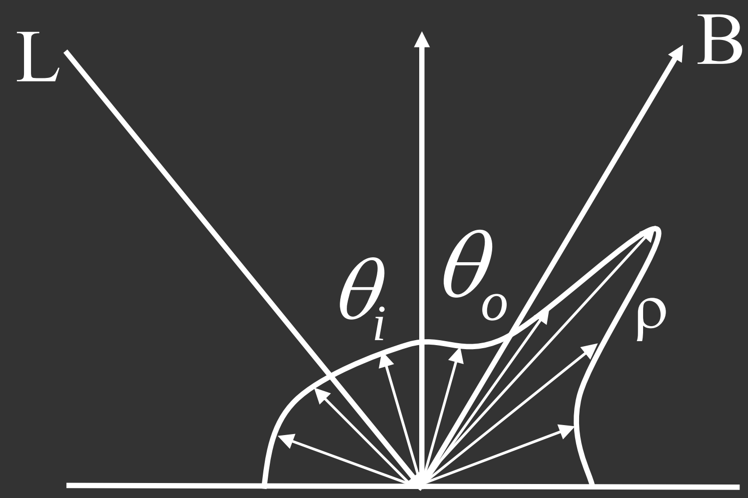
- Motivation for Inverse Rendering
- *Theory of Reflection as Convolution*
- Signal Processing Framework
- Spherical Harmonic Identities

Environment Maps

Miller and Hoffman, 1984

Later, Greene 86, Cabral 87, 99,…

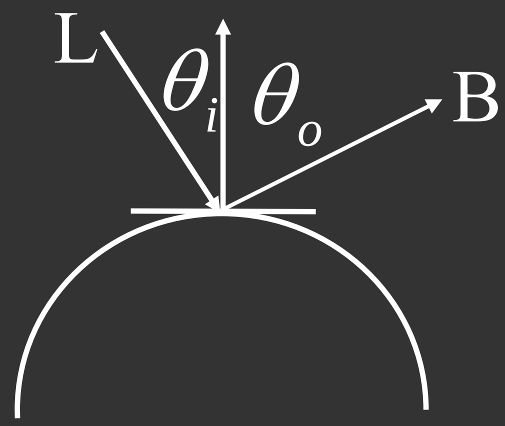
Reflection

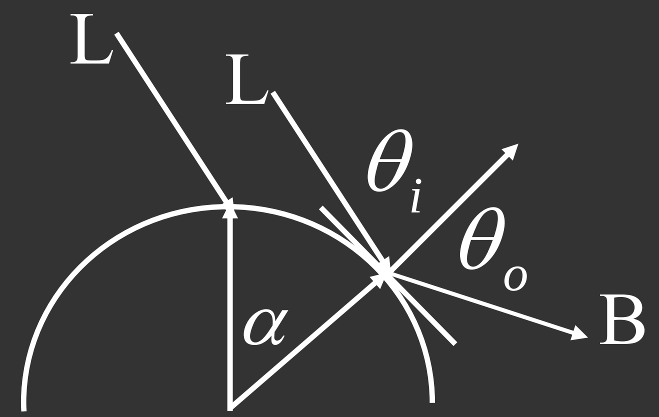


$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\ d\theta_i$$

Reflected Light Field **Lighting** **BRDF**

Reflection as Convolution (2D)

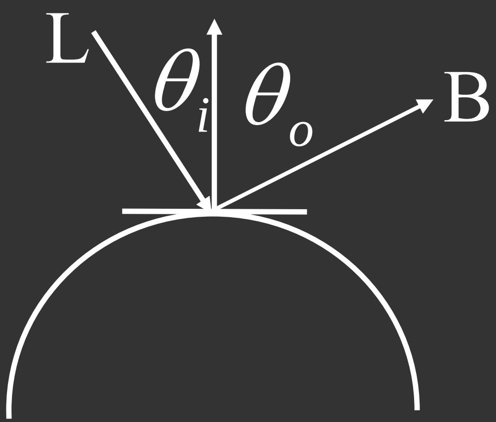


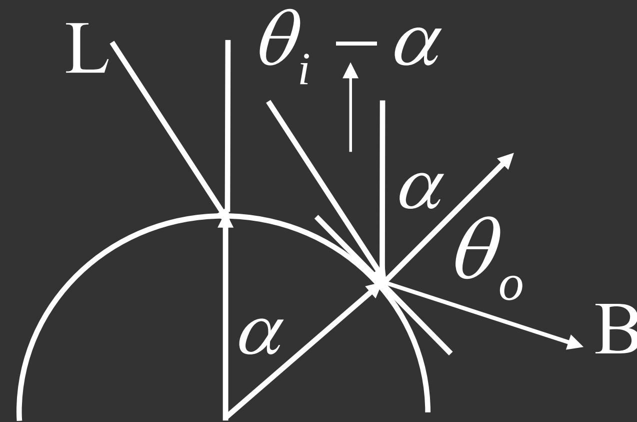


$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\, d\theta_i$$

Reflected Light Field **Lighting** **BRDF**

Reflection as Convolution (2D)



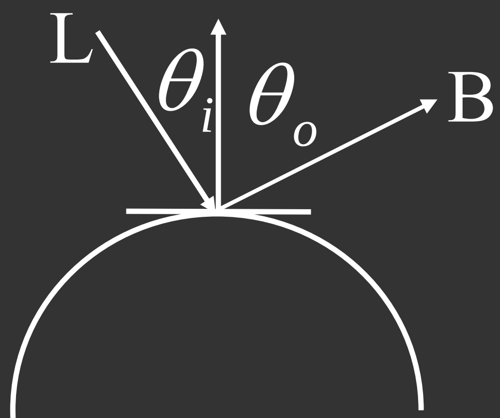


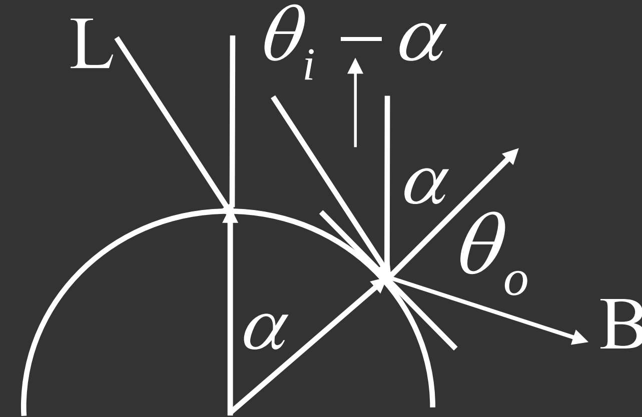
$$B(\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i)\ \rho(\theta_i, \theta_o)\, d\theta_i$$

Reflected Light Field **Lighting** **BRDF**

$$B(\alpha, \theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \rho(\theta_i, \theta_o)\, d\theta_i$$

Reflection as Convolution (2D)





$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha)\rho(\theta_i,\theta_o)\, d\theta_i$$

$$B = L \otimes \rho$$

Fourier analysis

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

Spatial: integral

Frequency: product

R. Ramamoorthi and P. Hanrahan "Analysis of Planar Light Fields from Homogeneous Convex Curved Surfaces under Distant Illumination" SPIE Photonics West 2001: Human Vision and Electronic Imaging VI pp 195-208

Related Work

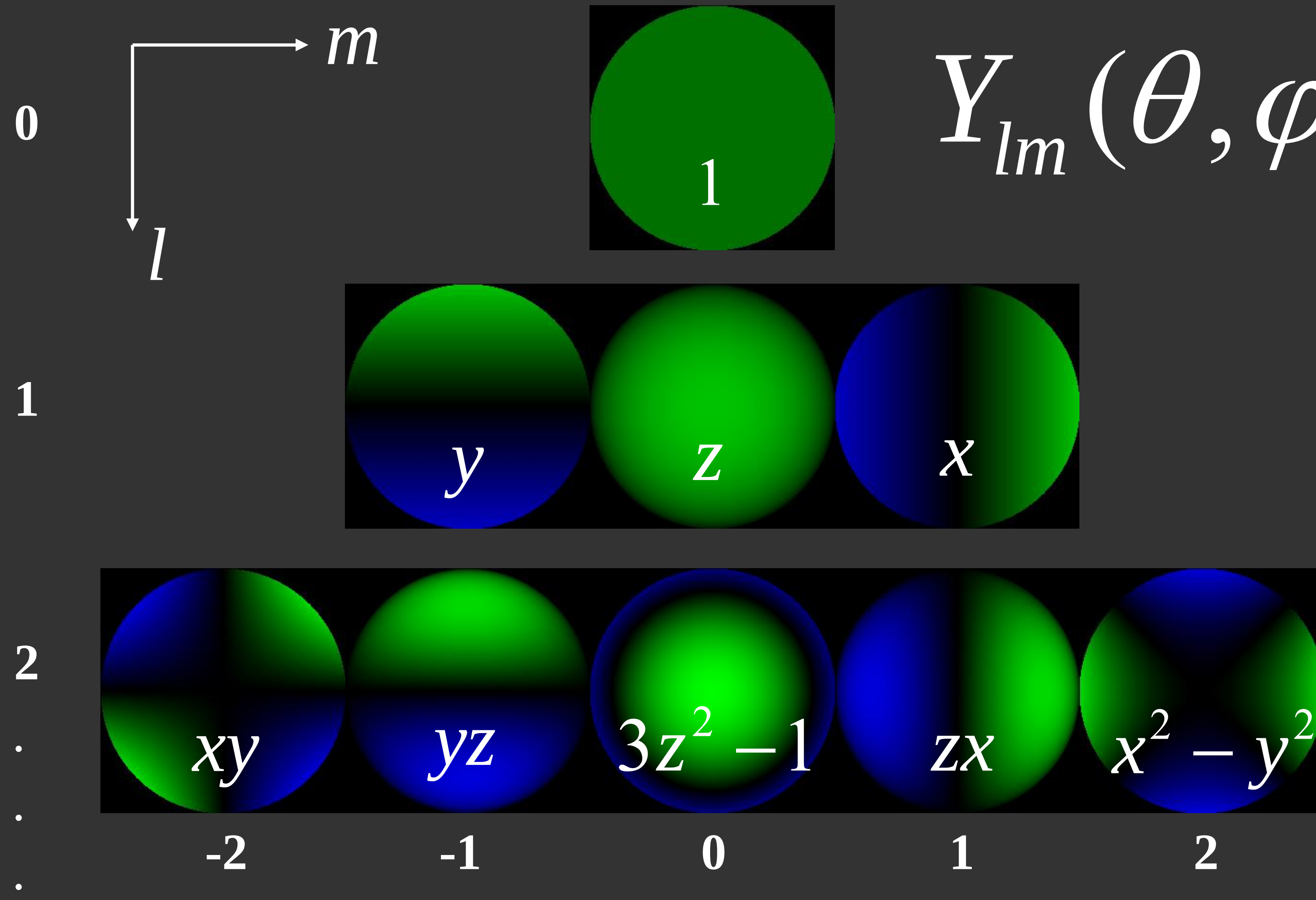
- Qualitative observation of reflection as convolution: Miller & Hoffman 84, Greene 86, Cabral et al. 87,99
- Reflection as frequency-space operator: D'Zmura 91
- Lambertian reflection is convolution: Basri Jacobs 01

Our Contributions

- Explicitly derive frequency-space convolution formula
- Formal quantitative analysis in general 3D case

Spherical Harmonics

$$Y_{lm}(\theta,\varphi)$$



Spherical Harmonic Analysis

2D:

$$B(\alpha,\theta_o) = \int_{-\pi/2}^{\pi/2} L(\theta_i - \alpha) \quad \rho(\theta_i,\theta_o) \quad d\theta_i$$

$$B_{l,p} = 2\pi L_l \rho_{l,p}$$

3D:

$$B(\alpha,\beta,\theta_o,\varphi_o) = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L(R_{\alpha,\beta}[\theta_i,\varphi_i])\rho(\theta_i,\varphi_i,\theta_o,\varphi_o)\, d\theta_i d\varphi_i$$

$$B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq}$$

Outline

- Motivation for Inverse Rendering
- Theory of Reflection as Convolution
- *Signal Processing Framework*
- Spherical Harmonic Identities

Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Insights: Signal Processing

Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Filter is Delta function : Output = Signal

Mirror BRDF : Image = Lighting [Miller and Hoffman 84]

Image courtesy Paul Debevec

Insights: Signal Processing

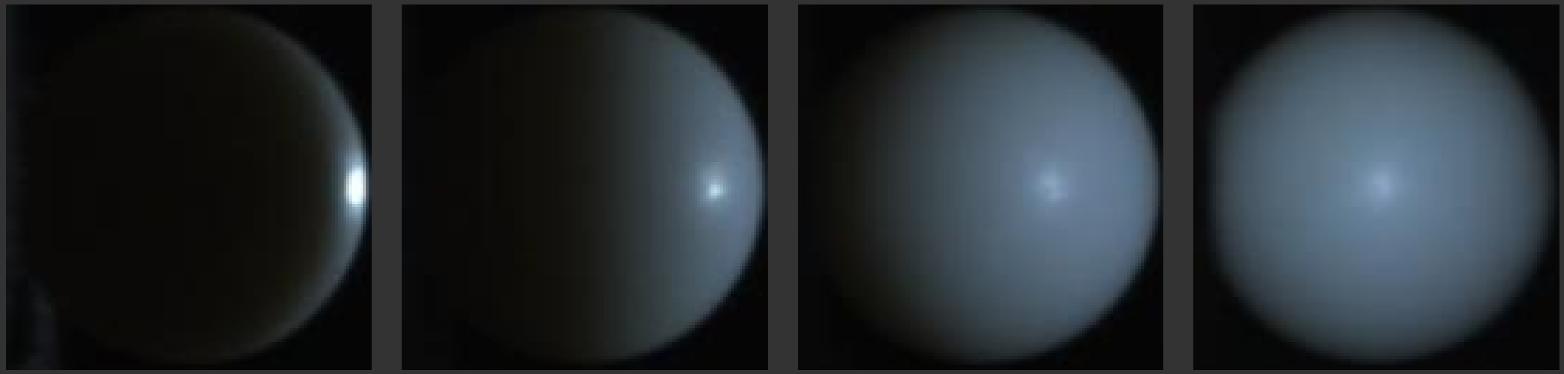
Signal processing framework for reflection

- Light is the signal
- BRDF is the filter
- Reflection on a curved surface is convolution

Signal is Delta function : Output = Filter

Point Light Source : Images = BRDF

[Marschner et al. 00]

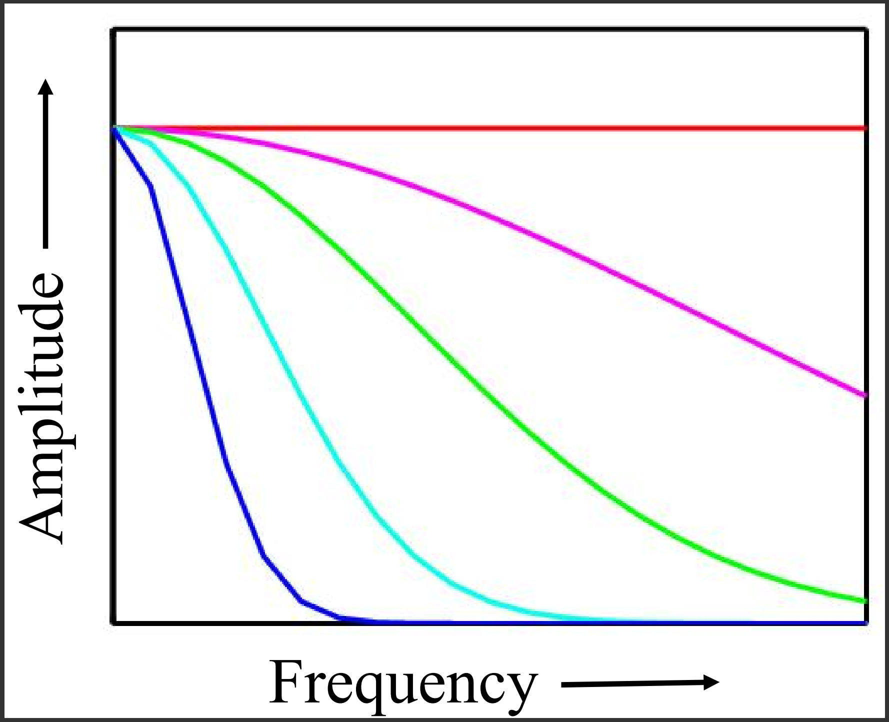


Phong, Microfacet Models

Mirror

Roughness

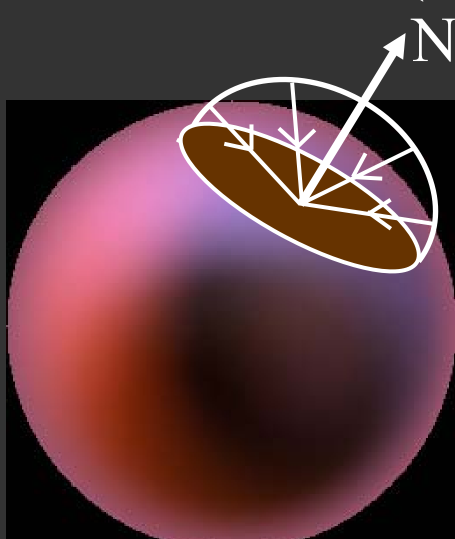
Illumination estimation
ill-posed for rough surfaces



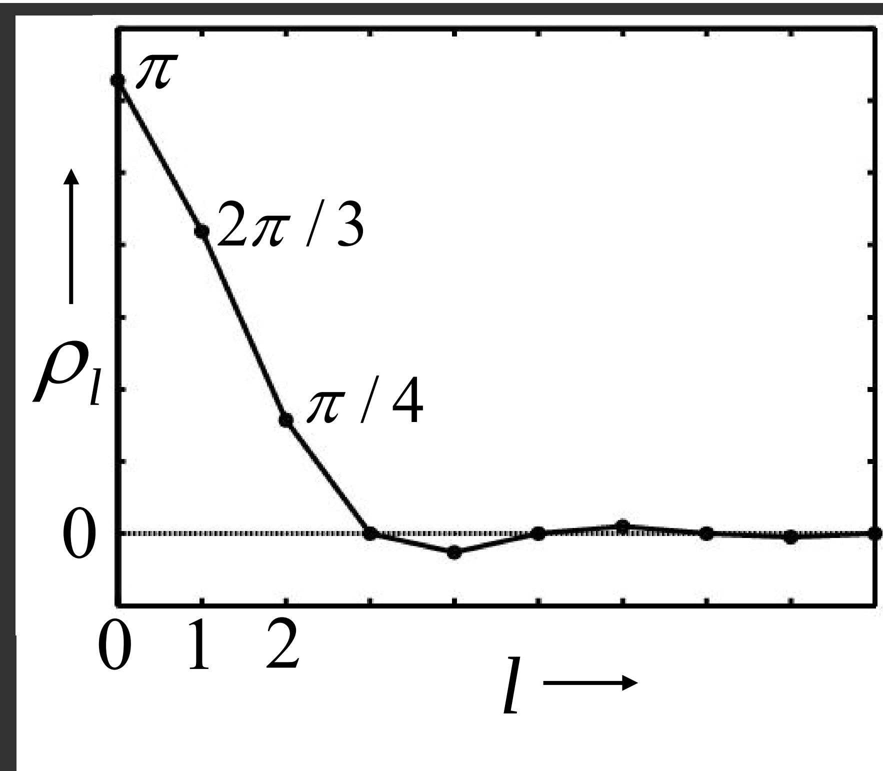
Analytic formulae in R. Ramamoorthi and P. Hanrahan
"A Signal-Processing Framework for Inverse Rendering"
SIGGRAPH 2001 pp 117-128

Lambertian

Incident radiance (mirror sphere)



Irradiance (Lambertian)



$$A_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)}\left[\frac{l!}{2^l\left(\frac{l}{2}!\right)^2}\right] \quad l \text{ even}$$

R. Ramamoorthi and P. Hanrahan "On the Relationship between Radiance and Irradiance: Determining the Illumination from Images of a Convex Lambertian Object" Journal of the Optical Society of America A 18(10) Oct 2001 pp 2448-2459

R. Basri and D. Jacobs "Lambertian Reflectance and Linear Subspaces" ICCV 2001 pp 383-390

Inverse Lighting

Given: B,ρ find L

$$B = L \otimes \rho$$

$$B_{lm,pq} = \Lambda_l L_{lm} \rho_{lq,pq}$$

$$L_{lm} = \frac{1}{\Lambda_l} \frac{B_{lm,pq}}{\rho_{lq,pq}}$$

Well-posed unless denominator vanishes

- BRDF should contain high frequencies : Sharp highlights
- Diffuse reflectors low pass filters: Inverse lighting ill-posed

Inverse BRDF

Given: B,L find ρ

$$\rho_{lq,pq} = \frac{1}{\Lambda_l} \frac{B_{lm,pq}}{L_{lm}}$$

Well-posed unless L_{lm} vanishes

- Lighting should have sharp features (point sources, edges)
- BRDF estimation ill-conditioned for soft lighting

Directional Source

Area source

Same BRDF

Practical Example

3 photographs of cat sculpture

- Complex unknown illumination
- Geometry known
- Estimate microfacet BRDF *and* distant lighting

New View, Lighting

Photograph

Rendering

Ramamoorthi and Hanrahan, 01c

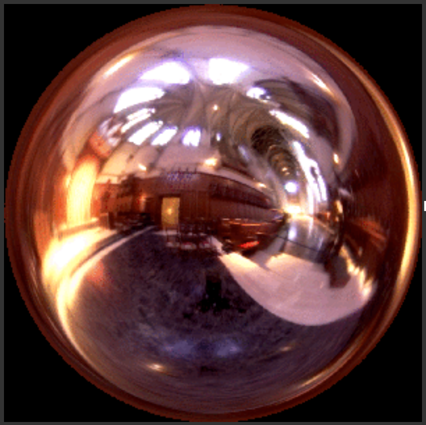
Outline

- Motivation for Inverse Rendering
- Theory of Reflection as Convolution
- *Signal Processing Framework*
  - *Other Applications*
- Spherical Harmonic Identities

Computing Irradiance

- Classically, hemispherical integral for each pixel

Incident Radiance



Irradiance

- Lambertian surface is like a low pass filter
- Frequency-space analysis (spherical harmonics)

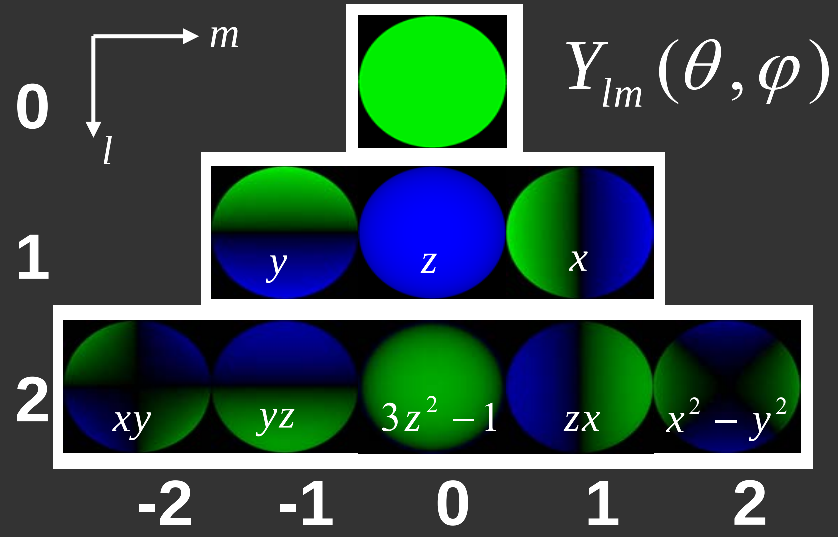
9 Parameter Approximation

Exact image

Order 2
9 terms

RMS Error = 1%

For any illumination, average error < 2% [Basri, Jacobs 01]



Ramamoorthi and Hanrahan 01b

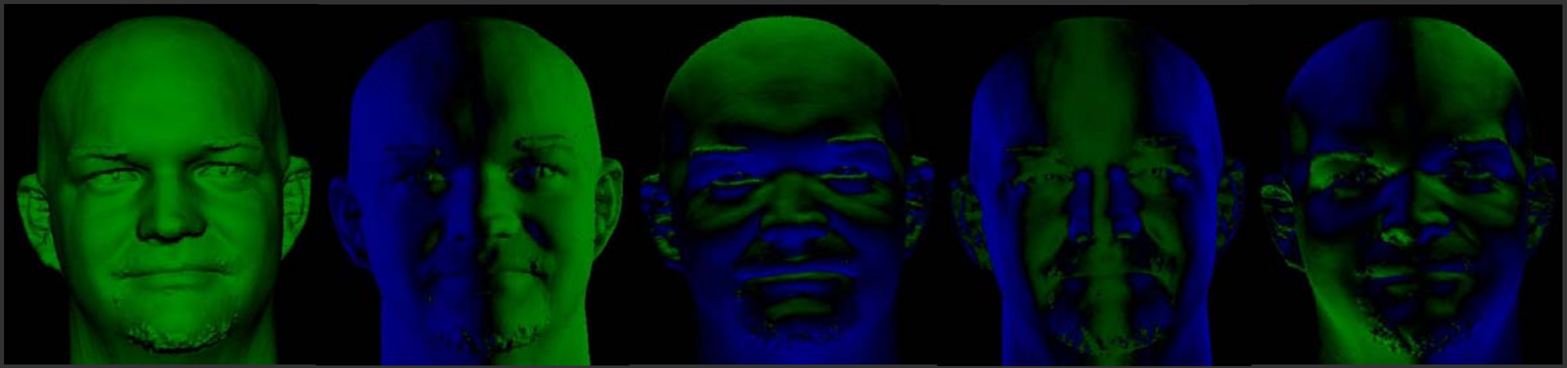
Real-Time Rendering

Motivation: Interactive rendering with natural illumination and realistic, measured materials

Computer Vision Complex Illumination

Low Dimensional Subspace

- Lighting Insensitive Recognition (Basri and Jacobs 01, Lee et al. 01, Ramamoorthi 02, …)



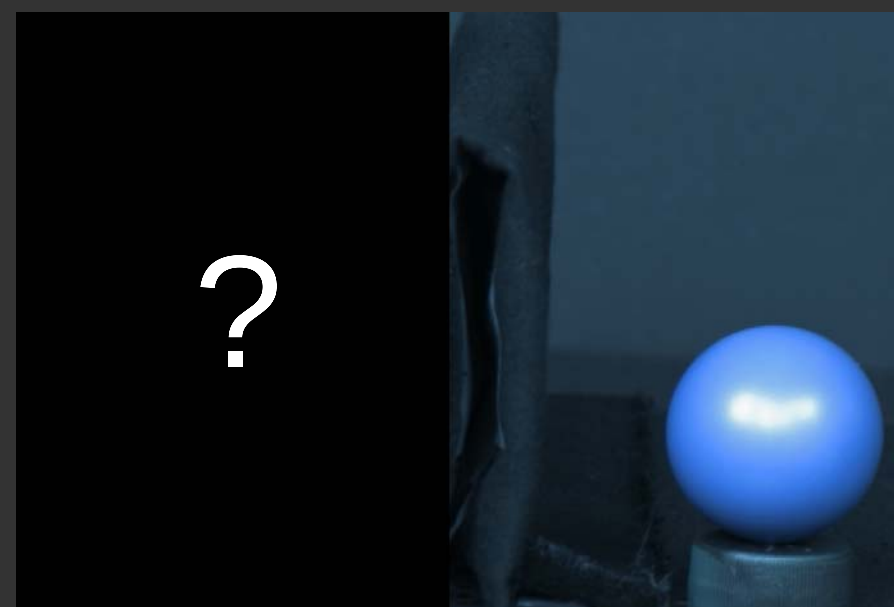
- Photometric stereo, shape acquisition

Outline

- Motivation for Inverse Rendering
- Theory of Reflection as Convolution
- Signal Processing Framework
- *Spherical Harmonic Identities*

Direct Object Relighting

Unknown
Lighting



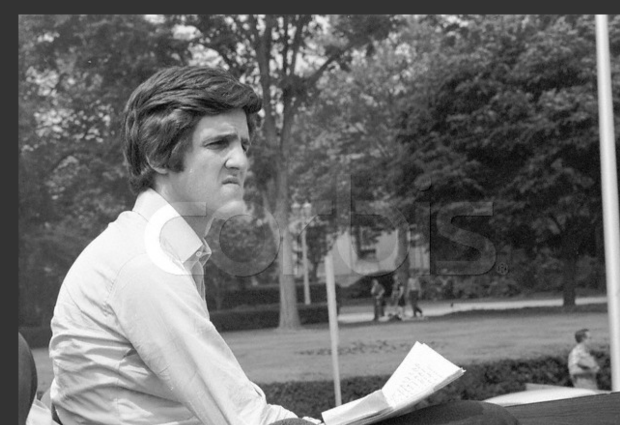
Checking Image Consistency

- Easy to tamper / splice images
  - Image processing software widely available
- In news reporting and other applications
  - Need to detect tampering or photomontage
  - Verify image consistency
- Try to check consistency of lighting, shading

Checking Image Consistency


Fonda Speaks To Vietnam Veterans At Anti-War Rally

Actress And Anti-War Activist Jane Fonda Speaks to a crowd of Vietnam Veterans as Activist and former Vietnam Vet John Kerry (LEFT) listens and prepares to speak next concerning the war in Vietnam (AP Photo)



Two Objects – Two Lightings

Material 1 | Material 2

Lighting 1

$$B_{lm}^{11} = A_l^1 L_{lm}^1$$

$$B_{lm}^{12} = A_l^2 L_{lm}^1$$

Lighting 2

$$B_{lm}^{21} = A_l^1 L_{lm}^2$$

$$B_{lm}^{22} = A_l^2 L_{lm}^2$$

Two Objects – Two Lightings

Material 1 | Material 2

Lighting 1

$$B_{lm}^{11} = A_l^1 L_{lm}^1$$

$$B_{lm}^{12} = A_l^2 L_{lm}^1$$

$$B_{lm}^{11} B_{lm}^{22} = A_l^1 A_l^2 L_{lm}^1 L_{lm}^2$$

Lighting 2

$$B_{lm}^{21} = A_l^1 L_{lm}^2$$

$$B_{lm}^{22} = A_l^2 L_{lm}^2$$

Two Objects – Two Lightings

Material 1

Material 2

Lighting 1

$$B_{lm}^{11} = A_l^1 L_{lm}^1$$

$$B_{lm}^{12} = A_l^2 L_{lm}^1$$

$$B_{lm}^{11} B_{lm}^{22} = A_l^1 A_l^2 L_{lm}^1 L_{lm}^2$$

$$B_{lm}^{12} B_{lm}^{21} = A_l^1 A_l^2 L_{lm}^1 L_{lm}^2$$

Lighting 2

$$B_{lm}^{21} = A_l^1 L_{lm}^2$$

$$B_{lm}^{22} = A_l^2 L_{lm}^2$$

Two Objects – Two Lightings

Material 1

Material 2

Lighting 1

$B_{lm}^{11} = A_l^1 L_{lm}^1$

$B_{lm}^{12} = A_l^2 L_{lm}^1$

$$B_{lm}^{11} B_{lm}^{22} = A_l^1 A_l^2 L_{lm}^1 L_{lm}^2$$

$$B_{lm}^{12} B_{lm}^{21} = A_l^1 A_l^2 L_{lm}^1 L_{lm}^2$$

Lighting 2

$B_{lm}^{21} = A_l^1 L_{lm}^2$

$B_{lm}^{22} = A_l^2 L_{lm}^2$

$$\boxed{B_{lm}^{11} B_{lm}^{22} = B_{lm}^{12} B_{lm}^{21}}$$

Independent of Lighting and BRDF

Image Estimation Framework

	Material 1	Material 2
Lighting 1	 B_{lm}^{11}	 B_{lm}^{12}
Lighting 2	 B_{lm}^{21}	? B_{lm}^{22}

$$B_{lm}^{11} B_{lm}^{22} = B_{lm}^{12} B_{lm}^{21}$$

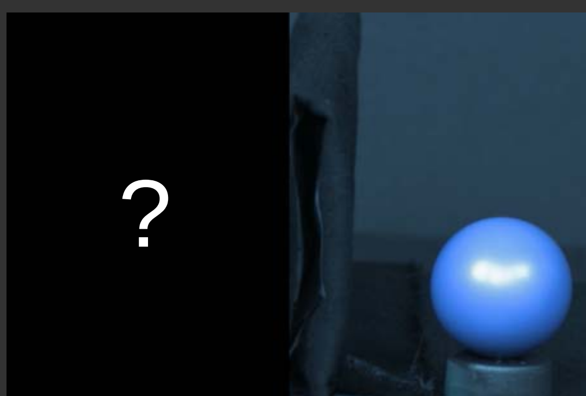
$$B_{lm}^{22} = \frac{B_{lm}^{12} B_{lm}^{21}}{B_{lm}^{11}}$$

Image Estimation

Lighting 1

Object 1 Object 2

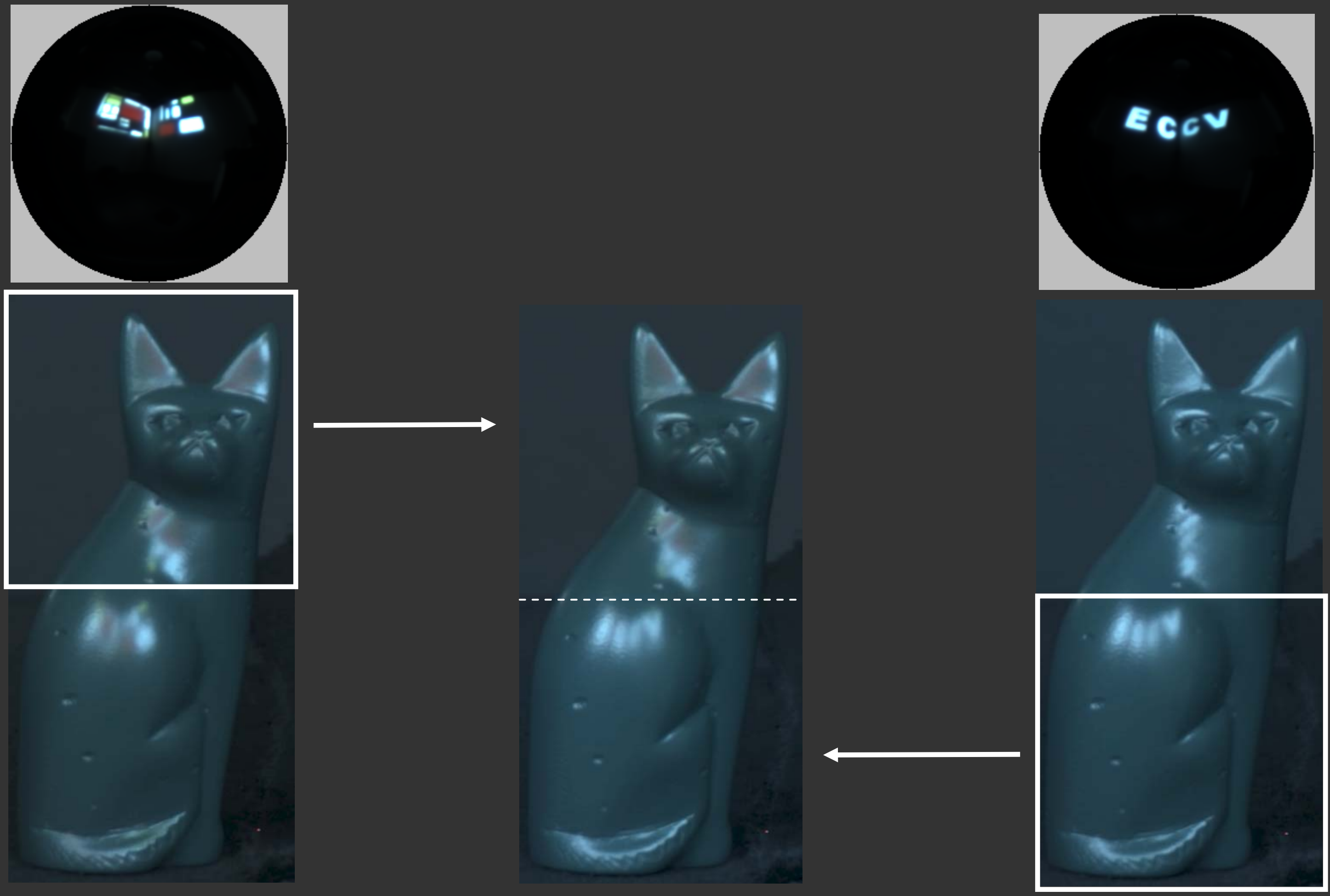
Lighting 2



Our Method Actual

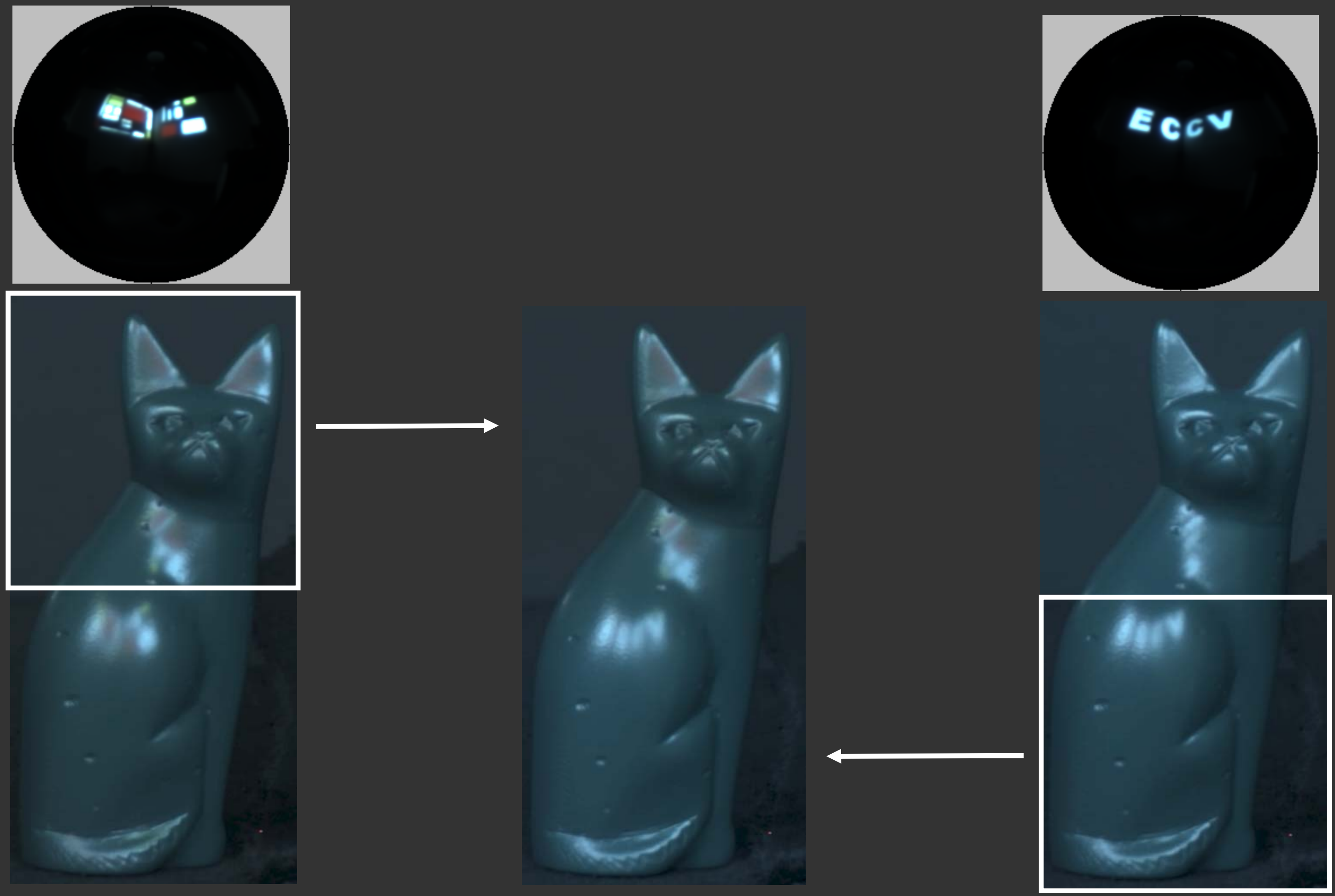
No BRDF and lighting known or estimated

Image Consistency Checking



Spliced Image

Image Consistency Checking

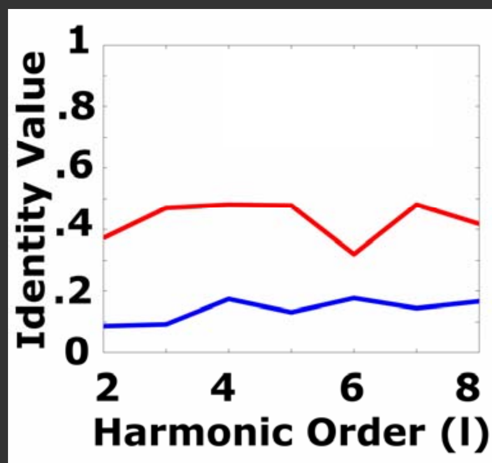


Spliced Image

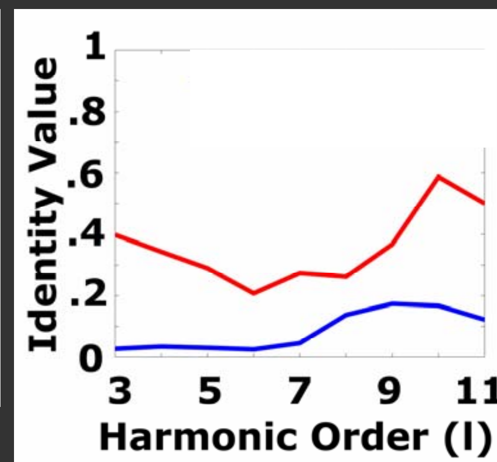
Image Consistency Checking

Tampered Cat

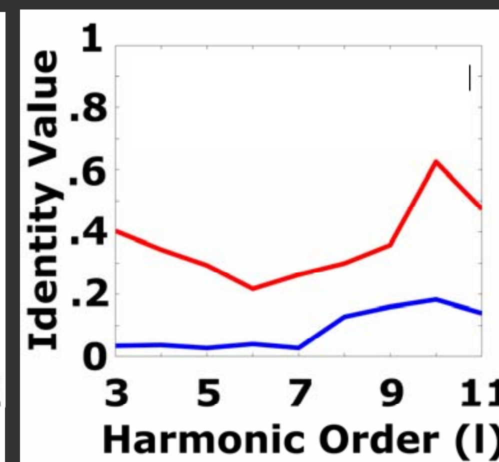
Tampered Cat — Untampered Cat



Single Image Identity diffuse + specular case



Two Lightings – Same Reflectance Identity



Two Materials – Two Lightings identity

Signal Processing for Appearance

Signal Processing widely applicable visual appearance

- Convolution relation for cast shadows [Soler and Sillion 98, Ramamoorthi et al. 04]
- Convolution with glows for participating media (mist, fog, haze) [Sun et al. 05]
- Signal-Processing analysis of light field and reflectance [Chai et al. 00, Zickler et al. 06]
- Triple Product Integrals [Ng et al. 04]
- First Order Analysis [Ramamoorthi et al. 06]

Acknowledgements

- Collaborators
  - Dhruv Mahajan
  - Brian Curless
  - Pat Hanrahan
- Funding: NSF, Sloan Foundation, ONR

http://www.cs.columbia.edu/~ravir

Questions