DESIGN OF MAGNETIC FIELDS FOR FUSION EXPERIMENTS

Allen H. Boozer
Columbia University

I. Plasma Confinement with Magnetic Fields

1. Why of interest:

For fusion energy $(D + T \rightarrow n + He)$ need a plasma of *1 atm.* pressure, *$10^8 K$* temperature, with an energy confinement time $\tau_E = 3 sec$. $(p_{DT} = \frac{3}{2} nT / \tau_E)$ Cannot be confined by solid materials!

2. How confined: $\vec{\nabla} p = \vec{j} \times \vec{B}$ with $\mu_0 \vec{j} = \vec{\nabla} \times \vec{B}$.

3. Issues:

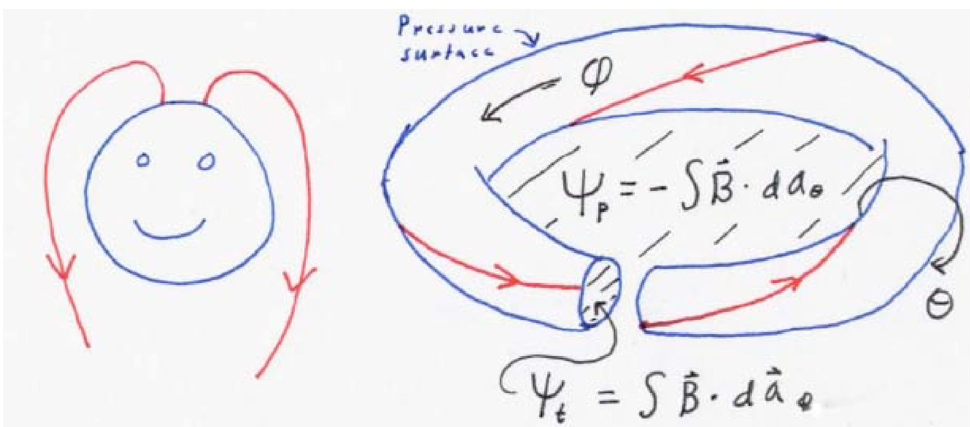
a. Efficient use of the magnetic field, $\beta = 2\mu_0 p / B^2$.
b. Transport consistent with the required τ_E.

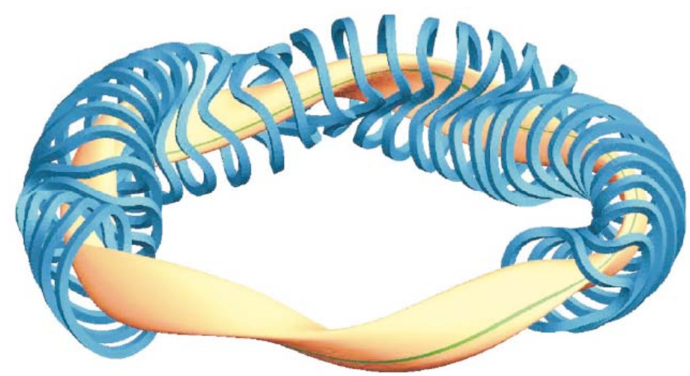
II. Plasma Equilibrium and Optimization

1. $\vec{\nabla} p = \vec{j} \times \vec{B}$ implies $\vec{B} \cdot \vec{\nabla} p = 0$.
Lines of $\vec{B}$ must lie in constant-p surfaces, which must be toroidal.

2. Required externally produced $\vec{B}$ implied by giving outermost plasma surface $\vec{x}_s(\theta, \varphi)$.

3. Plasma can be optimized by varying R and Z of (R,φ,Z) cylindrical coordinates $\vec{x}_s(\theta,\varphi) = R(\theta,\varphi)\hat{R}(\varphi) + Z(\theta,\varphi)\hat{Z}$.





III. Role of External Coils

1. Provide toroidal magnetic flux $\psi_t \equiv \int \vec{B} \cdot d\vec{a}_\varphi$. *(simple)*

2. Ensure $\vec{B} \cdot \hat{n} = \vec{B}_{coils} \cdot \hat{n} + \vec{B}_{plasma} \cdot \hat{n} = 0$ on plasma surface.

IV. Issues in Coil Design

1. Coil problem: Produce required $\vec{B}_{coils} \cdot \hat{n}$ on plasma surface.

2. Plasma problem: Avoid resonant fields, which destroy the magnetic surfaces even at low amplitude, $\Delta B / B \approx 10^{-4}$.

IV (1). Issues in Coil Design (Coil problem)

A specified $\vec{B}_{coils} \cdot \hat{n}$ cannot be produced by distant coils!

About 50 independent distributions $b(\theta,\varphi) \equiv \vec{B}_{coils} \cdot \hat{n}$ allowed.

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Suppose $B_r(a,\theta) = \sum_m B_m \cos(m\theta)$ is specified on the *r=a* surface of *(r,θ,z)* cylindrical coordinates. If the coils are on the *r=b* surface,

$$B_r(b,\theta) = \sum \left(\frac{b}{a}\right)^{m-1} B_m \cos(m\theta), \text{ but}$$

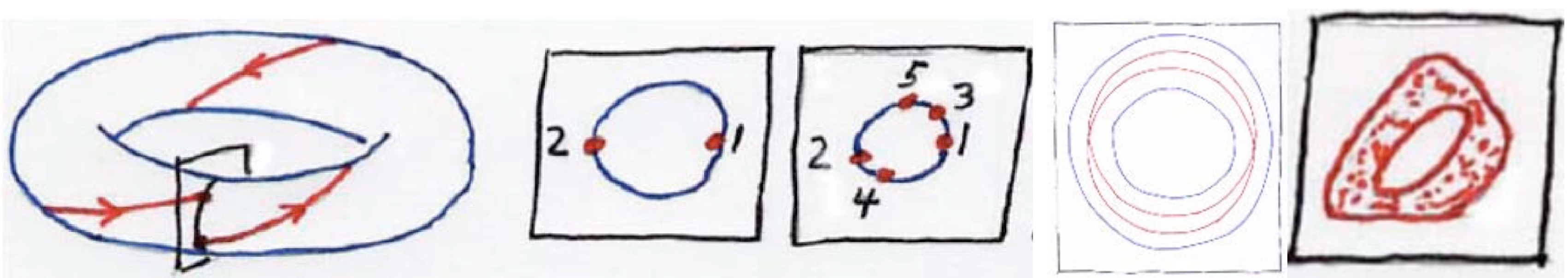
$$\pi B_m = \text{Real}\left[\int_0^{2\pi} B_r(a,\theta) e^{im\theta} d\theta\right] \propto e^{-m\theta_I} \quad \text{as } m \to \infty, \text{ so}$$

$\left(\frac{b}{a}\right)^{m-1} B_m \to e^{m\{\ln(b/a)-\theta_I\}}$; Coils cannot create field if $b/a \geq \exp(\theta_I)$.

IV (2). Issues in Coil Design (Plasma problem)

Pressure (or magnetic) surfaces in the plasma can be destroyed by resonant fields--even at low amplitude, $\Delta B / B \approx 10^{-4}$.

Magnetic field lines are the trajectories of a $1\frac{1}{2}$ degree of freedom Hamiltonian mechanics problem. The constants of the motion of the Hamiltonian are the magnetic surfaces, which are broken in $\sqrt{\Delta B / B}$ order and can become stochastic.



V. Perturbed Equilibrium Calculations

Codes exist that perturb the plasma shape $\vec{x}_s(\theta,\varphi)$ keeping the pressure profile and the topology of the $\vec{B}$-lines fixed.

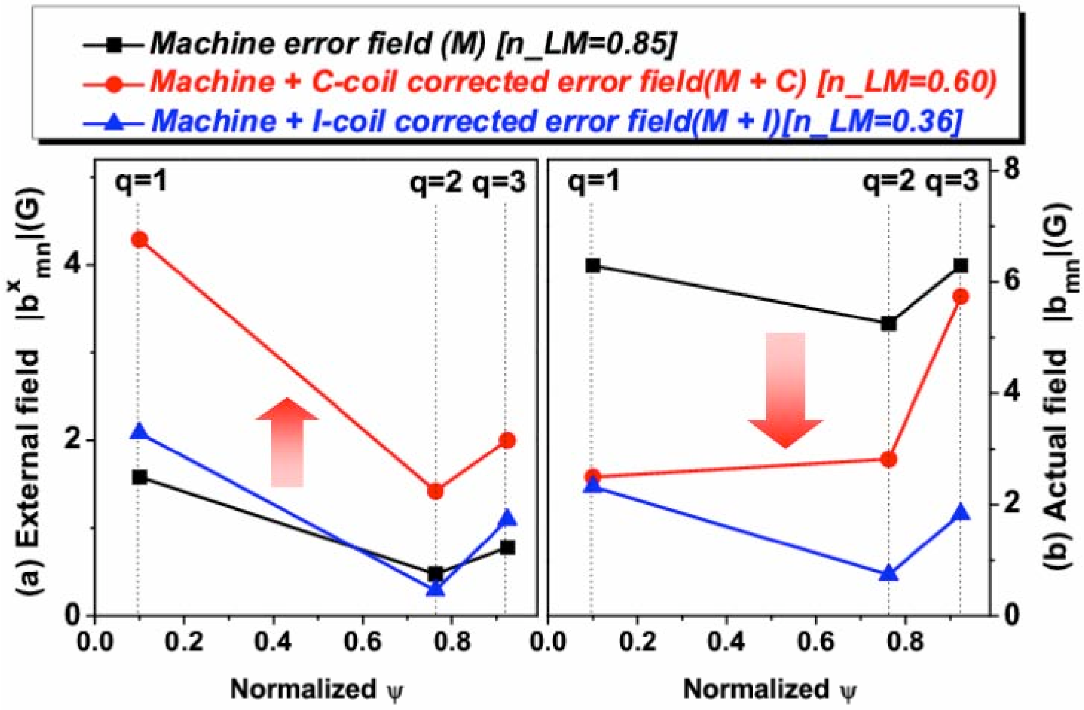
On surfaces on which the $\vec{B}$-lines close on themselves, codes find the surface current required to preserve topology of $\vec{B}$-lines.

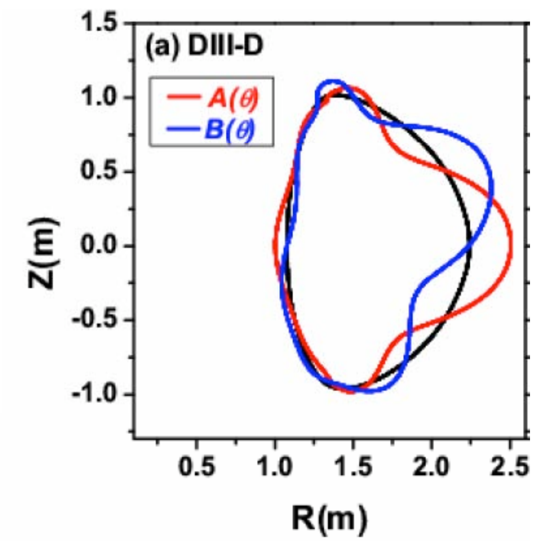
The strength of this surface current gives the drive for the breaking of the magnetic surfaces to form islands and stochastic regions.

These codes were written for stability calculations. J-K Park has modified the DCON code to calculate the plasma response to a given $\delta\vec{B}_{coils}\cdot\hat{n}$.

VI. Most Sensitive Perturbations (J-K Park)

A specific $b(\theta,\varphi) \equiv \delta\vec{B}_{coils} \cdot \hat{n}$ exists to which the plasma is most sensitive for the destruction of magnetic surfaces. Does not resemble the resonant Fourier harmonic!





VII. Strategy for Improving Coils and Plasmas

J. Nührenberg and P. Merkel (1990's) developed a strategy.

1. Start with a plasma configuration that appears desirable.
2. Calculate new plasma equilibria with new shapes.
3. Move in the directions that make better coils and plasma.

Major limitation is the loss of pressure surfaces as coils improved.

Codes (like J-K Park's) will make it possible to avoid perturbations that drive surface break up. Should allow improved coils. Perturbed equilibria also quickly determine plasma improvements.

Optimization must be in the space of about 50 distributions $b(\theta,\varphi) \equiv \vec{B}_{coils} \cdot \hat{n}$ that distant coils efficiently produce.