Inverse Problems and Imaging

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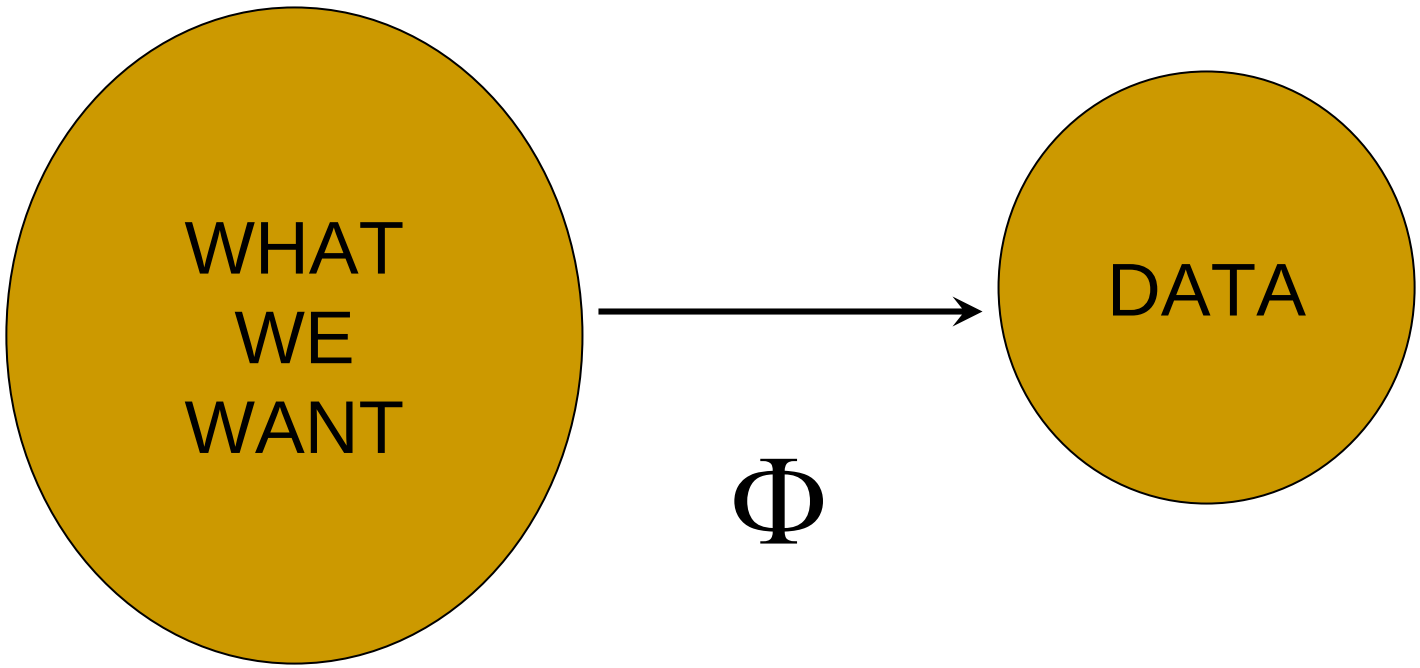
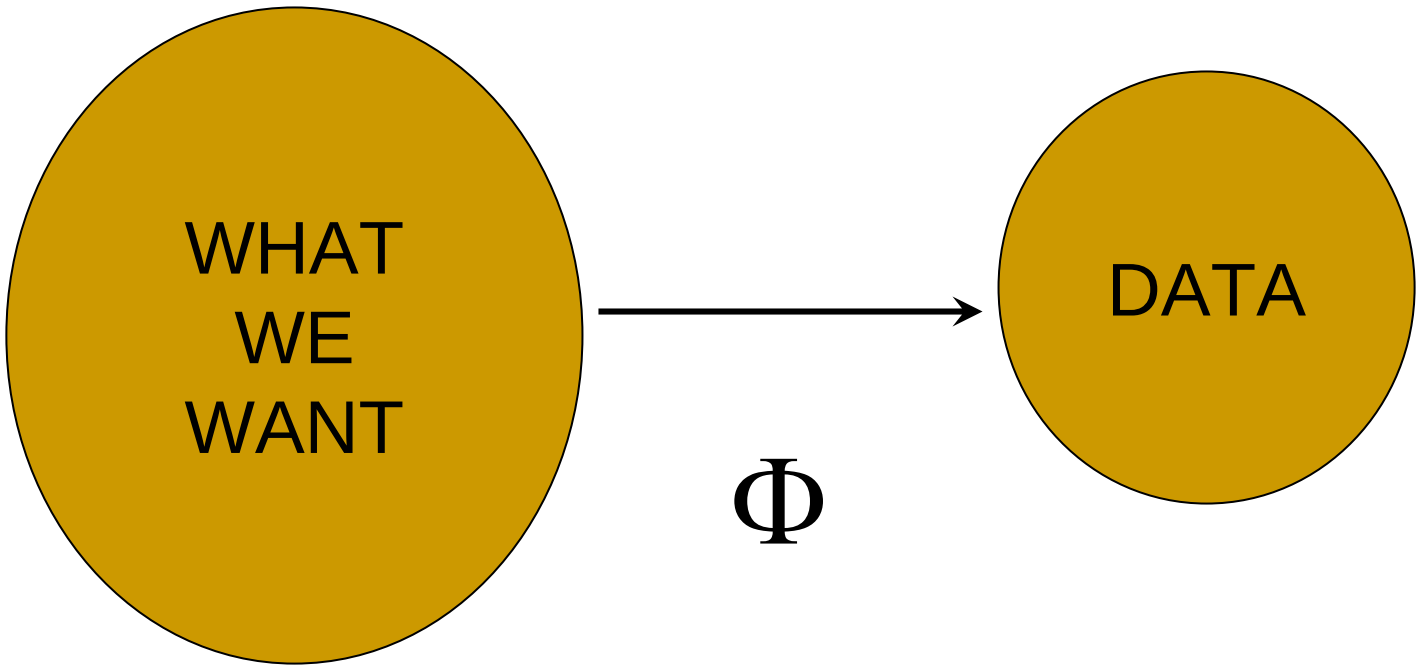
COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

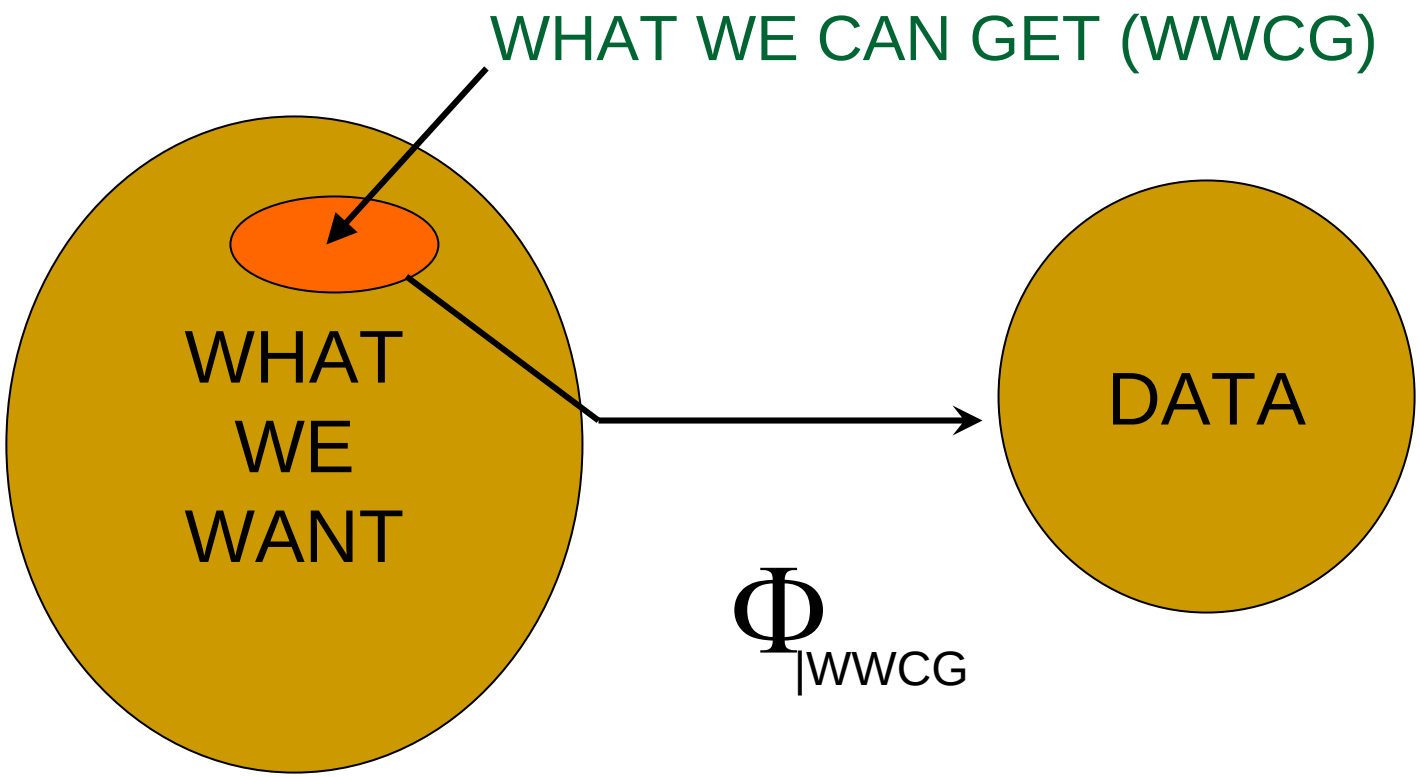
Inverse Problems and Imaging



Problem: Φ is often SMOOTHING



Solution: Simplify the problem



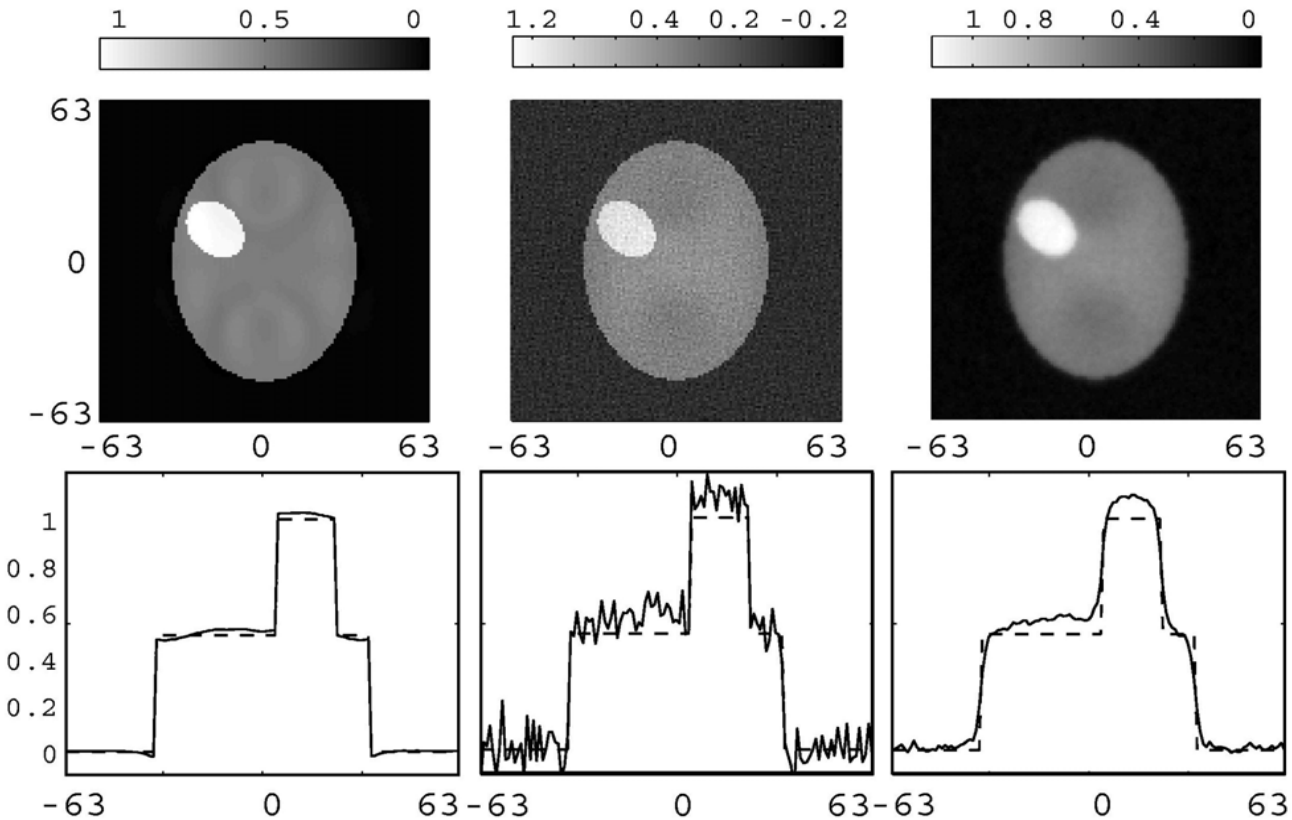
Best compromise

- WWCG has to be practically interesting
- $\Phi_{|WWCG}$ has to have bounded inverse
- *Or DATA set has to be augmented (by new experiments)*

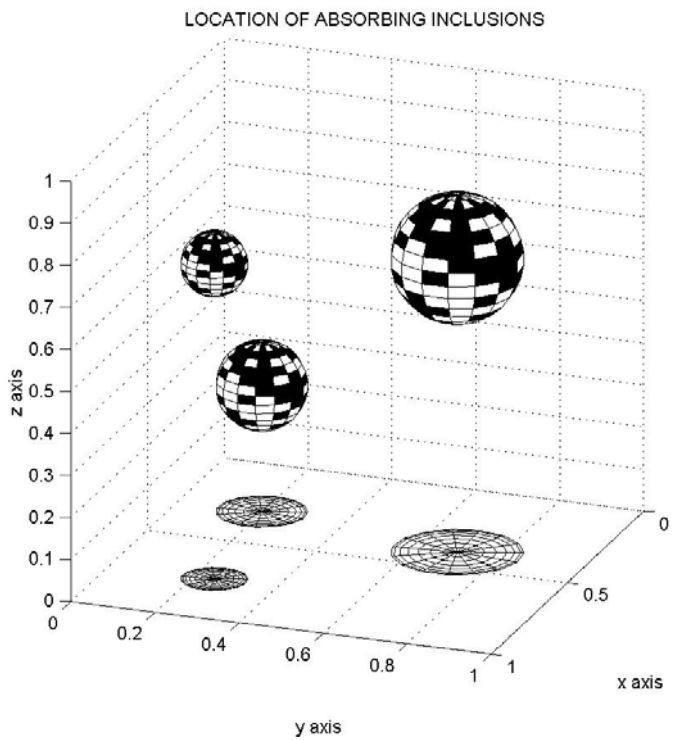
Rough classification

- Well-posed
  - Inverse Fourier transform
    - MRI
- Mildly ill-posed
  - Integral along curves
    - X-ray tomography
    - Travel Time Tomography
- Severely ill-posed
  - Smoothing maps
    - Optical and electrical impedance tomography; deblurring; anything involving a potential equation.

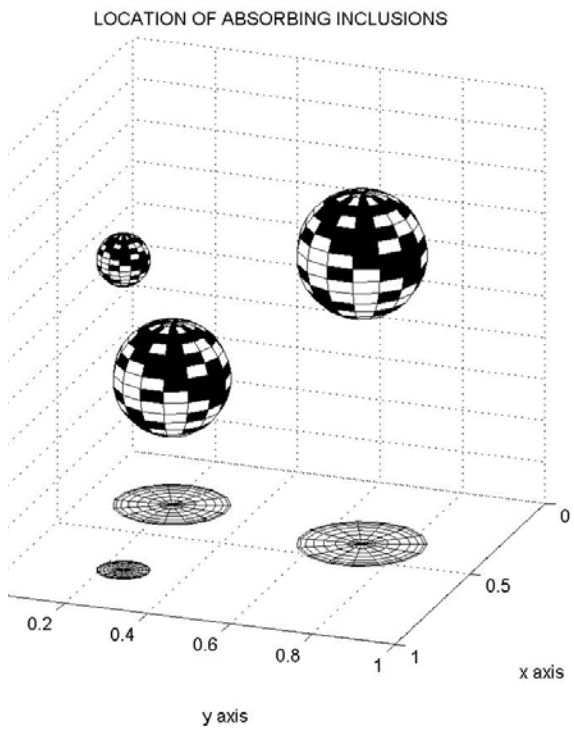
Simplification by Regularization



Simplification by Parameterization



NOISE-FREE DATA



NOISY DATA

Other simplifications

- Optimal discretizations
- Combination of modalities (multi-modality medical imaging)
- Use a more macroscopic forward model

Once a forward model is chosen:

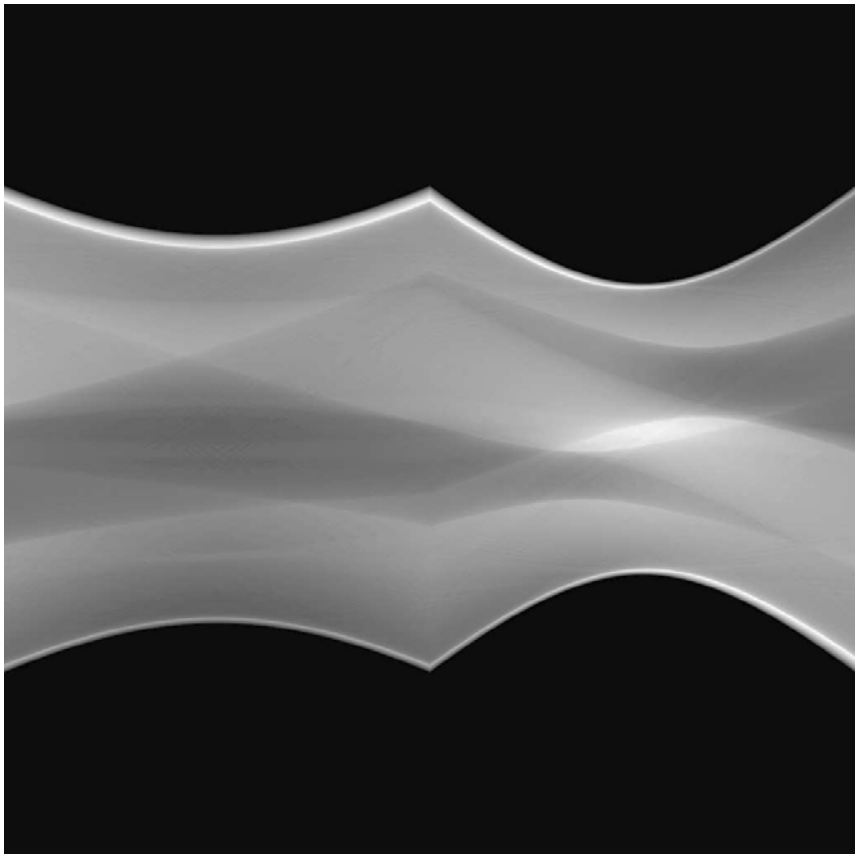
- Optimization/minimization issues
- Numerical Simulations (usually costly)
- Noise Model:
  - 1) Deterministic
  - 2) Statistical: Law [noise] $\rightarrow$ Law [WWCG]

Inverse Problems versus Imaging

- Inverse Problem: Get a first image from experimental data
- Imaging: Improve on that (noisy) image by further constraining what we want

- *One should be able to do better in ONE step (unanswered problem)*

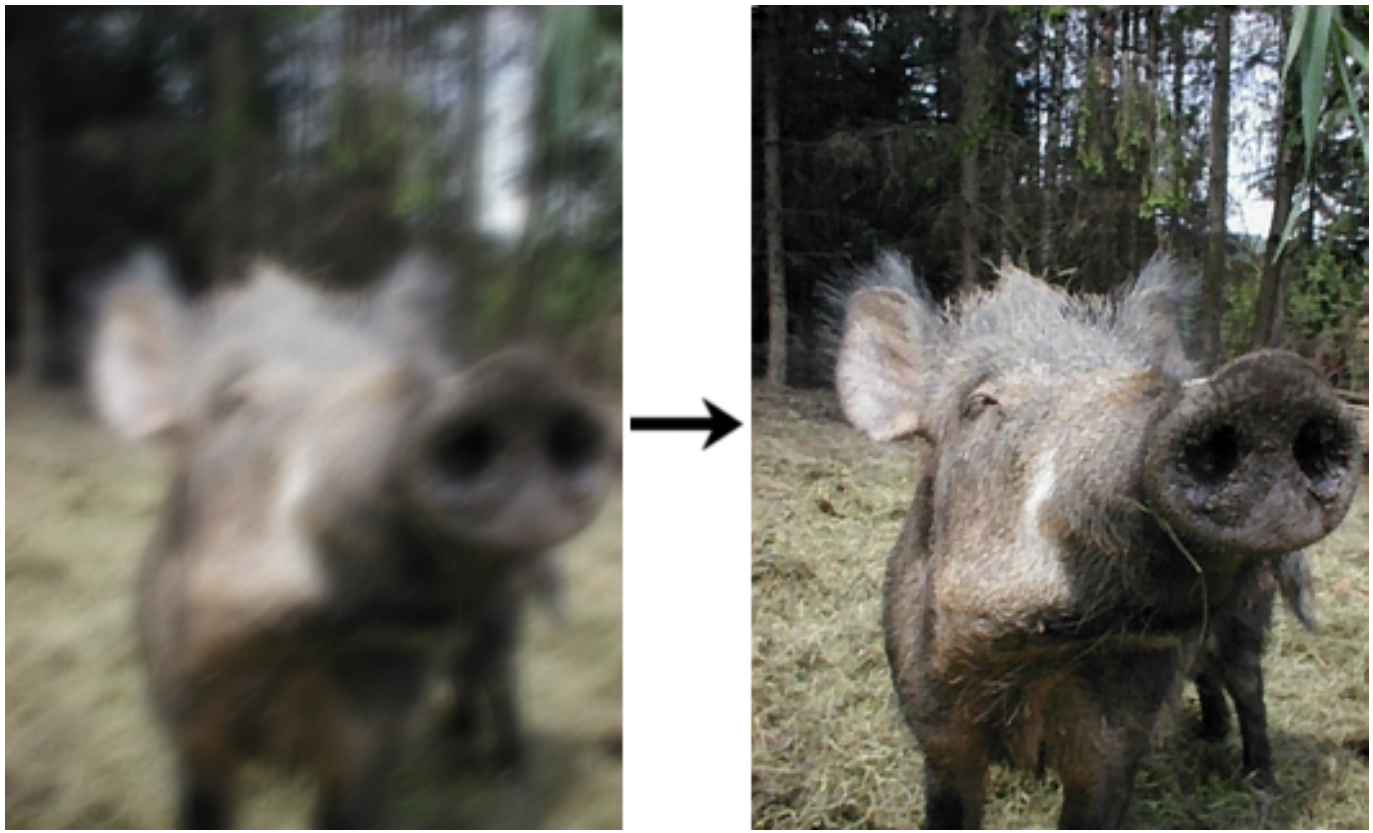
Typical Inverse Problem



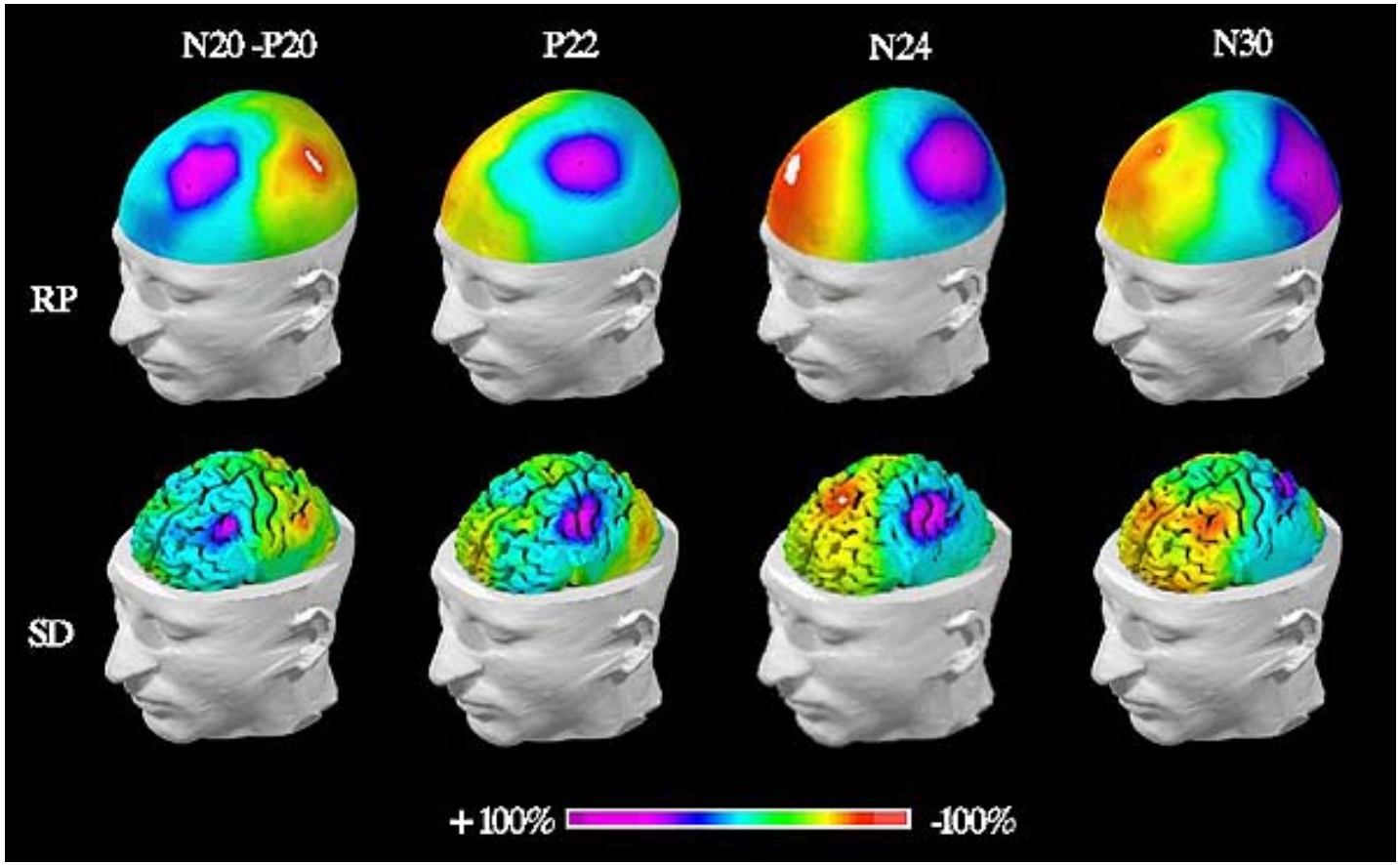
RADON TRANSFORM

RECONSTRUCTED PHANTOM

Typical Imaging problem



Typical combination



TOP: RECONSTRUCTED POTENTIAL
BOTTOM: SPATIALLY DEBLURRED POTENTIAL

IPs are ubiquitous. Speakers from:

- Biological sciences
- Computer graphics
- Earth-Science
- Financial Engineering
- Material Science
- Medical Imaging
- Plasma Physics

THANK YOU FOR YOUR PARTICIPATION

ACKNOWLEDGMENT:

DEPARTMENT OF APPLIED PHYSICS AND APPLIED MATHEMATICS